Precalc Warm Up # 4-3

A test over logarithms was given to a precalc class. It was then given a month later, and then a month after that, and so on, to see if kids were forgetting. The average test score is given by

$s(t) = 90 - 20 \log (t+1)$ where s(t) is the score (as a percent) after t months

What was the average score when students were first tested? After 6 months? After 2 years?

How long before the average score is 50%?

Effective Yield:

The actual percentage paid in a year.

Compare the yield of investing at 5.2% annual interest rate...

$A(t) = P(1 + r)^t$

$A(1) = P(1 + 0.052)^1$

$A(1) = P(1.052)$

5.2%

Effective Yield if Compounded Daily?

$A(t) = P(1 + \frac{r}{n})^{nt}$

$A(1) = P(1 + \frac{0.052}{365})^{365(1)}$

$A(1) = P(1.0534)$

5.34%

HW Questions: p. 293

	Initial investment	Annual % rate	Effective yield	Time to double	Amount after 10 years
5.	$500				$1,292.85
9.	$5000		8.33%		

HW Questions: p. 293

$A = Pe^{rt}$

	Initial investment	Annual % rate	Effective yield	Time to double	Amount after 10 years
		9.5%	9.97%		
5.	$500				$1,292.85
9.	$5000	8%	8.33%		

$A = P\left[e^{\left(\frac{\ln 2.5857}{10}\right)}\right]$

$\frac{1,292.85}{500} = \frac{500e^{r(10)}}{500}$

$2.5857 = e^{10r}$

$\frac{\ln 2.5857}{10} = \frac{10r}{10}$

$A = P(1+r)$

$1+r = e^{\left(\frac{\ln(2.5857)}{10}\right)}$

$1+r \approx 1.0997$

$\approx 9.97\%$

$e^r = 1.0833$

$r = \ln(1.0833)$

In Exercises 13 and 14, determine the time necessary for $1000 to double if it is invested at interest rate r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.

13. $r = 11\%$

a) $A = P(1+r)^t$

$\frac{2000}{1000} = \frac{1000}{1000}(1+0.11)^t$

$2 = (1.11)^t$

$\frac{\ln 2}{\ln(1.11)} = \frac{t \ln(1.11)}{\ln(1.11)}$

$t \approx$

b) $A = P\left(1 + \frac{r}{12}\right)^{12t}$

$2 = \left(1 + \frac{0.11}{12}\right)^{12t}$

$\frac{\ln 2}{12 \ln\left(\frac{12.011}{12}\right)} = \frac{12t \ln\left(\frac{12.011}{12}\right)}{12 \ln\left(\frac{12.011}{12}\right)}$

$t \approx$

21. The population P of a city is given by

$$P = 105{,}300e^{0.015t}$$

where t is the time in years, with $t = 0$ corresponding to 1985. According to this model, in what year will the city have a population of 150,000?

	Isotope	Half-life (years)	Initial quantity	Amount after 1000 years	Amount after 10,000 years
25.	Ra^{226}	1,620	10 g		

$\frac{1}{2}C = Ce^{k(1620)}$

$\frac{1}{2} = e^{1620k}$

$\frac{\ln 0.5}{1620} = \frac{1620k}{1620}$

$y = 10e^{\left(\frac{\ln 0.5}{1620}\right)(1000)}$

$y \approx 6.52$

29.	Pu^{230}	24,360	2.1 g

$2.1 = Ce^{\left(\frac{\ln 0.5}{24,360}\right)(1000)}$

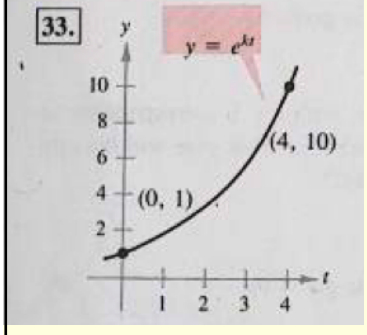
Using $\frac{1}{2}$ life

$\frac{1}{2} = e^{k(24,360)}$

$\frac{\ln 0.5}{24,360} = \frac{24,360k}{24,360}$

In Exercises 33–36, find the constant k such that the exponential function $y = Ce^{kt}$ passes through the given points on the graph.

33.



C = Initial Amount

C = 1 → from (0,1)

$y = (1)e^{kt}$

use (4,10)

$10 = e^{k(4)}$

Solve for k:

$\frac{\ln 10}{4} = \frac{4k}{4}$

37. The sales S (in thousands of units) of a new product after it has been on the market t years are given by

$S(t) = 100(1 - e^{kt})$.

(a) Find S as a function of t if 15,000 units have been sold after one year.

(b) How many units will be sold after five years?

a) $15 = 100(1 - e^{k(1)})$

Solve for k → $\frac{15}{100} = 1 - e^{k}$

$e^{k} = 0.85$

$k = \ln 0.85$

$S(t) = 100(1 - e^{(\ln 0.85)t})$

b) $S(5) = 100(1 - e^{5 \ln 0.85})$

39. A certain lake is stocked with 500 fish, and the fish population increases according to the logistics curve

$$p(t) = \frac{10{,}000}{1 + 10e^{-t/5}}$$

where t is measured in months.

(a) Find $p(5)$.

(b) After how many months will the fish population be 2000?

$2000 = \frac{10{,}000}{1 + 10e^{-t/5}}$

$1 + 10e^{-t/5} = \frac{10{,}000}{2{,}000}$

$10e^{-t/5} = 4$

$e^{-t/5} = 0.4$

$-\frac{t}{5} = \ln 0.4$

$-t = 5 \ln 0.4$

$t = -5 \ln 0.4$

$t \approx$

In Exercises 49 and 50, use **Newton's Law of Cooling**, which states that the rate of change in the temperature of an object is proportional to the difference between its temperature and the temperature of its environment. If $T(t)$ is the temperature of the object at time t in minutes, T_0 is the initial temperature, and T_e is the constant temperature of the environment, then

find first

$$T(t) = T_e + (T_0 - T_e)e^{-kt}.$$

T_0 $T(45)$

49. An object in a room at 70° F cools from 350° F to 150° F in 45 minutes. T_e

(a) Find the temperature of the object as a function of time.

(b) Find the temperature after it has cooled for one hour.

(c) Find the time necessary for the object to cool to 80° F.

a) $150 = 70 + (350 - 70)e^{-k(45)}$

$\frac{80}{280} = \frac{280e^{-45k}}{280}$

$\frac{2}{7} = e^{-45k}$

$\ln\left(\frac{2}{7}\right) = -45k$

$\frac{\ln\left(\frac{2}{7}\right)}{-45} = \frac{-45k}{-45}$

$k = -\frac{\ln\left(\frac{2}{7}\right)}{45}$

$T(t) = 70 + 280e^{\left(\frac{\ln\frac{2}{7}}{45}\right)t}$

b) plug in 60 for t

c) $80 = 70 + 280e^{(k)t}$ (k from above)

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50. A thermometer is taken from a room at 72° F to the outdoors, where the temperature is 20° F. The reading on the thermometer drops to 48° F after one minute. Determine the reading after five minutes.

$T_0 = 72°$

$T_e = 20°$

$t = 1$ min.

$T(1) = 48°$

$$T(t) = T_e + (T_0 - T_e)e^{-kt}$$

① plug in what you know and find k.

② Put k into your model (exact) → or store it

$k \approx 0.6190$

STO→

③ Use your model, plug in $t = 5$,

$T(5) \approx 22.35°$

Group Event Part 2

HW: PC book

p. 295 Chapter 4 Review

3, 19, 25, 31, 47, 49, 53

... and make sure you have checked the review WS key online!